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ABSTRACT

Although recent writings indicate the importance of personal contact structures in diffusing innovations and in determining perceptions, the internal contact structure of the school faculty remains unexamined. This study applies directed graph theory, a new branch of mathematics, to analyzing school contact structure. Sociometric data and perceptions of the school were collected from 48 elementary school faculties containing 1,121 people. Using directed graph theory, 12 indices of the contact structure were derived from the sociometric data and entered into canonical correlations and factor analyses with perceptions of the school, such as faculty innovativeness. Several significant relationships were obtained. (Author)

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APPLYING DIRECTED GRAPH THEORY TO FACULTY CONTACT STRUCTURE[1]

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Overview

Some recent theory and empirical studies stress the importance of personal contact for the diffusion of innovations. Relationships have been suggested between the individual's position in the social structure and his receipt of innovations (Becker, 1970) and between his position and his perceptions of the social structure (Rice and Mitchell, 1973). Yet the school contact structure remains unexamined. Part of the difficulty is methodological: a sociogram of any size is enormously complex. Analysis across schools is almost unknown. The approach in this study is to analyze the contact structure using a relatively new body of mathematics, directed graph theory (Harary, Norman, Cartright, 1965), which has not been extensively applied.

Directed graph theory describes the relationships between nodes in structural models as represented in a sociogram or a PERT chart. The power of the theory is that these relationships can be reduced to matrices which then can be subjected to matrix algebra operations. In this study an "adjacency" matrix was initially constructed for each school from which was derived the "reachability" and "distance" matrices for the school. From these matrices twelve indices were constructed to describe the contact structure. These included the average social distances among faculty, percentages of social contacts, the symmetry of relationships, and several other ratios. These became the independent variables.

Data were collected from the faculties of 48 elementary schools with combined faculties of 1,121[2]. The basic datum was a six-point personal-contact scale ranging from "do not know who he is" to "very well acquainted," which every faculty member filled out on every other faculty member in his school. Other data included several semantic differential scales asking for perceptions of the school, demographic data, and scales on various educational beliefs. The semantic differential scales included five-point ratings of both the school administration and school faculty on innovativeness, influencing school policy, resourcefulness, openness to new ideas, involvement, and planning curriculum.

In the analysis the sociometric data were dichotomized as required by directed graph theory. The dividing line was drawn between occasional contacts and frequent contacts, based on previous research. Separate analyses were done for individuals and for total-school structures. The analysis of individuals relates the position of the individual to his perception of the school while the analysis of total-school structure relates the characteristics of the contact structure

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as a whole to school climate. Parallel indices were derived from directed graph theory for both individuals and the total school although these indices were not exactly similar.

Several multivariate analyses were run with the contact-structure indices and perceptions of school climate as variables. Canonical correlations were run with twelve social-structure variables as predictor variables and twelve perceptions of the school (semantic differential scales) as criterion variables. Factor analyses were used for reducing the number of variables for entry into further canonical correlations.

Some results are promising. There do seem to be empirical relationships between the contact structure of the school and its perceived characteristics. An unexpected finding is that the school-contact structure is 96 percent "reachable." That is, nearly every teacher in the school can reach every other teacher, mainly through first-generation contacts, and the remainder through no more than two intermediary contacts, at least in schools of this type and size. The numbers of intermediaries the teacher must pass through do seem to make a difference, but these seem to be few in number. Directed graph theory offers fresh insights into the contact structure and a promising approach to such analysis.

The Structural Variables (Predictors)

Twelve variables were chosen to describe the faculty-contact structure of the schools. The relationships among 50 faculty members represents a complex sociogram indeed and one too complex to interpret visually with any sophistication. Nor is it possible to relate the contact structure to other variables of interest. The basic sociometric data were supplied by asking 1,121 teachers in 48 schools to fill out a scale on every other teacher in the school. The scale ranged from "Do not know who he/she is" to "Very well acquainted/see both at school and outside school." Directed graph theory demands a dichotomous relation so the scale was dichotomized between "Somewhat acquainted/talk occasionally at school" at the low side and "Well acquainted, talk frequently but only at school" on the upper side. This variable separated the well acquainted from the rest. Some variables were also defined in terms of "Do not know who he/she is" plus "Know who he/she is but have never spoken"--in other words, "Do not know or talk to" versus "Know slightly or well."

Twelve variables were defined from these scales. The definitions of what to define are somewhat arbitrary since only "centrality" has been empirically related to perception variables. The twelve structure variables defined to describe the social structure were the following ratios:

1. $\frac{\text{(Number of "well-acquainted" choices given)}}{\text{(The number that could have been given)}}$

2. $\frac{\text{(Number of well-acquainted choices received)}}{\text{(Number possible to receive)}}$

3. $\frac{\text{(The symmetric relations) [well-acquainted choices reciprocated]}}{\text{(Number possible to give)}}$

4. $\frac{\text{(Symmetric relations)}}{\text{(Number of well-acquainted choices given)}}$

5. $\frac{\text{(Number of not-acquainted choices received)}}{\text{(Number possible)}}$

6. $\frac{\text{(Number of well-acquainted choices given)}}{\text{(Number of well-acquainted choices received)}} \times \frac{\text{(Symmetric relations)}}{\text{(Relations possible)}}$

7. Mean maximum distance required to reach all possible people

8. Mean maximum distance required to be reached by someone

9. Mean average distance reached = $\frac{\text{(Total distance to reach all)}}{\text{(Number of persons subject may reach)}}$

10. Mean average distance received = $\frac{\text{(Total distance from all)}}{\text{(Number of persons who may reach subject)}}$

11. $\frac{\text{(Number of persons whom subject may reach)}}{\text{(Number of persons who may reach subject)}}$

12. $\frac{\text{(Number of persons subject may reach)}}{\text{(Number possible to reach)}}$

Obviously while mathematically precise, those characteristics of the social system are not easily defined in common language. We chose where we could variable definitions used in previous studies and otherwise defined what seemed reasonable. Many other variables could be defined. These are relatively easy to obtain from the adjacency matrix, the reachability matrix, and the distance matrix for each school structure. These are arrived at through matrix algebra operations, and we will not go into those procedures which are defined in directed graph theory. All are derivable, however, from the basic adjacency matrix which describes who is talking to whom.

The means and standard deviations from these variables are presented below:

Variable	Mean	Standard Deviation
1	.48	.27
2	.48	.19
3	.33	.20
4	.69	.23
5	.052	.09
6	.38	.28
7	2.2	.79
8	2.6	.78
9	1.5	.45
10	1.5	.34
11	1.1	1.3
12	.96	.16

The mean size of each school faculty was 28 with a standard deviation of 11.

A number of interesting characteristics emerge from inspecting this descriptive data. Of the total faculty, the subjects say they are well acquainted with half (variable 1). Half the faculty say they are well acquainted with the subject (naturally variable 2). Of all the symmetric relations, i.e., reciprocated acquaintances it is possible to have, 33% of the choices are reciprocal (variable 3). Of all the well-acquainted choices made by the subjects, 69% are reciprocated (variable 4). Half the relationships consist of not knowing or not talking to the other people (variable 5).

The mean maximum distance required to reach all people in a sequence is 2.2 people (variable 7). The mean maximum distance required to be reached by everyone is 2.6 people in sequence (variable 8). Of those the subject knows the total average distance is only 1.5 people as opposed to reaching everyone (variable 9). Of those who know him the total distance to reach the subject is also 1.5 persons (variable 10). The number of persons the subject may reach as a ratio to the number who can reach him is about even (variable 11).

Perhaps the most interesting feature is the number of people of the total structure that the subject can reach (i.e., knows someone well who knows someone well, etc.). Fully 96% of the faculty can be reached by everyone (variable 12). The faculty structure is eminently "reachable" by extended contacts through people one knows well.

The picture that emerges of these elementary schools with a mean faculty of 28 is one of a close social structure in which, if one does not know everyone else, he knows someone who does. Fully one-third of all the relationships are reciprocated selections, and almost 70% of the well-acquainted relationships are reciprocated. Everyone can be "reached," i.e., communicated with through a well-acquainted relationship, by only a few people. Reaching the people he does not know or talk to adds only one person to the chain of communication. One can only conclude that the contact structure is highly integrated and ideas will travel very rapidly through them if frequency and span of contact are the criteria. The schools are reachable regardless of faculty size. Information will get out rapidly to everyone with a minimum of distortion since there is so much overlap in the communication structure.

The Perception Variables (Criterion)

Twelve perceptions of the professional climate of the school were collected from each teacher. Each teacher was asked to rate both the teachers and administration on six five-point semantic differential scales. The scales were these:

Resourceful - Not resourceful
Open to new ideas - Closed to new ideas
Involved - Unconcerned
Plan curriculum - Don't plan curriculum
Influence educational policy - Don't influence educational policy
Innovative - Rigid

Rating both teachers and administrators on all six scales yields twelve variables of the teachers' perceptions of the school. Ratings for the scales hovered around "4" out of a possible "5" with a standard deviation of about one. There was not much difference between ratings for teachers and ratings for administrators although that hypothesis was not formally tested.

Canonical Correlations on the New Variables

The twelve structural variables were then related to the twelve perception variables in canonical correlations. The results were given below:

Variate	Eigenvalue	Correlation	Wilks Lambda	Chi-Square	Df
1	0.1131	0.3363	0.7458	325	156
2	0.0462	0.2149	0.8409	192	132
3	0.0361	0.1900	0.8816	140	110

Only the first canonical variate has a correlation coefficient high enough to interpret. It is statistically significant but questionable because of the high lambda, which indicates that the significance may be a function of sample size. Nonetheless, for the first variate the predictor and criterion weights are listed below:

Variable	Predictor Weights	Criterion Weights
1	-0.11	-0.09
2	0.21	-0.12
3	-0.14	-0.14
4	0.11	-0.05
5	0.24	-0.07
6	0.14	-0.04
7	-0.25	-0.18
8	-0.29	0.00
9	0.88	-0.24
10	0.40	0.15
11	0.02	-0.24
12	-0.68	-0.34
Size	-0.66	

The predictor variables most heavily weighted are mean average distance to reach all (variable 9) and (negatively) percent of persons reachable and school size. These are negatively related to the criterion variables of administrator innovativeness, administrative influence on school policy, and administrative involvement. In other words, the shorter the distance to reach everyone, the larger the percent that can be reached and the larger the school, the more administrators are described as being innovative, influential, and involved. Note: School size and shorter mean distances are independently related to the criteria.

To reduce the number of variables, the perception variables were reduced by a principal components analysis.

Variable	Factors 1	2	3
1	.70	-.43	-.05
2	.72	-.45	.07
3	.77	-.42	-.09
4	.62	-.23	-.29
5	.59	-.12	-.32
6	.63	-.34	.43
7	.76	.36	.03
8	.71	.36	.21
9	.74	.39	-.01
10	.51	.38	-.42
11	.59	.34	-.22
12	.59	.29	.57

The first factor loads heavily on all the positive components. Both teachers and administrators are perceived as resourceful, open to new ideas, involved, planning curriculum, influencing educational policy, and innovative. This is a general "change" factor.

The second factor loads negative on teachers and positive on administrators in every item--a "teacher-administrator" factor.

Factor 3 loads heavist on innovation for both teachers and administrators--an "innovative flexibility" factor. These account for 65% of the variance.

Variate	Eigenvalue	Correlation	Wilks Lambda	Chi-Square	Df
1	0.1414	0.3760	0.8478	183	39
2	0.0107	0.1036	0.9874	14	24
3	0.0019	0.0432	0.9981	2	11

The first variate is significant, but the high lambda indicates the significant chi-square may be due to sample size.

Variable	Predictor Weights	Criterion Weights	
1	0.16	Factor 1	0.91122
2	-0.80	Factor 2	0.94932
3	0.54	Factor 3	0.56446
4	-0.09		
5	-0.34		
6	-0.45		
7	0.19		
8	0.24		
9	-0.69		
10	-0.69		
11	-0.04		
12	-0.52		
13	0.72		

The "change" factor and the "administrator-teacher" factor of the perception variables are negatively weighted with the percent of well-acquainted choices received, average distance to reach all, and the average distance from which the subject can be reached. It is positively related to size, the percent of persons subject may reach, and the percent of symmetric relations. Both the closer the subject is to the total structure and the larger the school, the more he perceives the change elements.

Since administrators are positive and teachers negative on that factor, high rankings on teachers are positively associated with percent of choices recorded and average distances. They are negatively related to size, percent reachable, and symmetric relations.

Repeated Analysis with Schools as Units

Essentially the same analyses were run using the 48 schools as the units of analysis rather than the individuals. Also some of the variables were changed somewhat. The new order of variables was:

1. Size of school

2. $\frac{\text{(Percent of well-acquainted choices)}}{\text{Number possible } (N^2 - N)}$

3. $\frac{\text{(Total number of symmetric relations)}}{\text{Total possible } \left(\frac{N^2 - N}{2}\right)}$

4. $\frac{\text{(Total of } \textit{very} \text{ well-acquainted choices)}}{\text{(Number possible)}}$

5. $\frac{\text{(Total number of no-acquaintance choices)}}{\text{(Total possible)}}$

6.-12. Same as first list of variables except calculated on schools as units

The means and standard deviations are below:

Variable	Mean	Standard Deviation
1	23	10.3
2	.50	.11
3	.71	.22
4	.12	.06
5	.04	.05
6	.40	.11
7	2.19	.45
8	2.48	.57
9	1.49	.22
10	1.52	.21
11	1.05	.22
12	.96	.05

There are few differences in these variables and the first set. The mean is 23 instead of 28, indicating there are more smaller schools. There is also a big difference in symmetric relationships. More than 73% are reciprocal, indicating that in small schools symmetrical relationships are more frequent than in the total N where the symmetrical percentage is 33%. Variable 4 is an entirely new variable here--the highest category of contact which indicates that socializing occurs outside school as well as within. The percent of total contacts that are "very well acquainted" is 12%. This indicates that 38% of the well-acquainted category is the less intense contact. The rest of the variables are close to the original. Apparently small schools affect the symmetry of choices considerably but not the reach, distance, etc.

The perception variables were again analyzed into principal components. This time the analysis yielded two components. Two components had eigenvalues above 1 and explained 80% of the variance.

Variable	Factors 1	2
1	-0.84	-0.23
2	-0.82	-0.45
3	-0.88	-0.31
4	-0.90	-0.14
5	-0.89	-0.21
6	-0.83	-0.36
7	-0.87	0.22
8	-0.84	0.21
9	-0.83	0.44
10	-0.46	0.71
11	-0.76	0.45
12	-0.79	0.54

Again the factors were the same as the first two of the other p.c. analysis--a "change" and a "teacher-administrator" factor. Entering these two factors into canonical correlations with the second set of structural variables, the results were:

Variate	Eigenvalue	Correlation	Wilks Lambda	Chi-Square	Df
1	0.4712	0.6864	0.5143	26	24
2	0.0275	0.1659	0.9725	1	11

Neither of these canonical variates was significant with these degrees of freedom. No further analysis was attempted.

Conclusion

The picture of the faculty contact that emerges from these analyses is of a highly integrated informal communication network and of faculties in which everyone is at most a few people removed from everyone else. The personal-contact structure is eminently "reachable." We were quite surprised to find

little evidence of cliques, isolates, etc. Perhaps we did not ask the right questions.

The multivariate analyses contributed little to the findings because of their equivocal nature. The correlations were low and quite possibly significant only because of the large N. For what they are worth, the general "change" factor of perceptions was related as follows:

Positively	Negatively
Size of school Percent reachable	Average distance to reach
Size of school Percent reachable Percent symetric	Percent choices received Average distance to reach Average distance accessible

In other words, those who perceived more change orientation were those who were in larger schools, could reach more people, and had more symmetric relations. The greater the social distances, the more negatively they viewed the change prospects. This tends to tie change to a closer, more integrated social structure but one operating in larger schools. The administrator-teacher scale, with administrators high, was related to the same variables. Administrators were perceived as more change oriented under the above conditions and teachers the other way around. (The perceptions were by teachers.)

The study does demonstrate that there are important facets to the social structure other than the ubiquitous one of size. Most of the characteristics mentioned above would ordinarily be subsumed under "size" although they are conceptually distinct, possibly important, and work independently of the size variable. The specific variables we chose to look at were quite arbitrary. Many other structural variables could have been defined and, of course, any number of dependent variables. Others would no doubt be more important.

Nonetheless, in working with the structural variables, we have gained new and fresh insights into the school social structure. Directed graph theory forces us to be precise in our perspectives and offers a new analytical handle. From the glimpses we have had, we think it is very promising.

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